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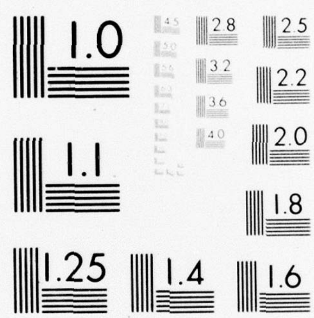

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SENSITIVITY APPROACH TO THE DUAL CONTROL PROBLEM*

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Abstract

A stochastic adaptive control problem that incorporates estimation cost considerations is formulated. A sensitivity index is introduced to represent the extimation cost. The control effort on estimation is distributed according to the accuracy required to achieve a given control objective when there is a bound on the estimation cost. The effect of the control on the sensitivity functions is used to influence the future uncertainties of the system. A dynamic feedback control algorithm is proposed that explicitly takes into account the accuracy of estimation and distributes the estimation effort in an optimal fashion.

1. Introduction

When a control law is to be designed for an uncertain system with unknown parameters, two roles of the control signal have to be considered [4]: one is the effect of the control signal on the estimation of the unknown plant parameters and/or state variables, and another is the attainment of the control objective. In general, these two roles of the control are conflicting [7]. The optimal solution to the dual control problem is achieved through the use of a closed-loop controller. The problem of getting the closed loop control law leads in general to a nonlinear problem and only approximate solutions can be obtained. Much effort has been made to obtain tractable methods of solution [1,2,10,12]; some are approximations to the optimal solution [10] and others use the conflicting characteristic of the dual property to constrain the control so as to achieve a short term control objective without exceeding a bound on the future covariances of the unknown quantities [1,13].

We propose a way of formulating a stochastic adaptive control problem that incorporates a cost assignment for the estimation effort and that automatically distributes the estimation budget in a rational way. In this approach a fixed budget for estimation is considered. The estimation effort is distributed according to the accuracy required to achieve a given control objective. This relationship between estimation and accuracy is based on the assumption that greater accuracy in the estimation of an unknown quantity implies a greater cost. Parameters to which the state of the system is more sensitive require more accurate estimation than those whose effect on the state is less significant. We will represent the fixed budget for estimation by a sensitivity constraint which is related to the estimation cost in the following fashion. It has been shown [7] that for time invariant systems the maximization of a sensitivity criterion [7,8] is equivalent to the maximization of the Fisher information matrix. We will use this fact to explicitly modify the control to affect the future uncertainties of the system. When we look at the estimation problem, the Cramer-Rao inequality [11] gives us a lower bound on the covariance of the estimate of the unknown parameters. We will use the influence of the control on the sensitivity of the system to decrease this lower bound.

The exact computation of the state sensitivity functions represents an infinite dimensional system when the feedback includes the sensitivity terms; due to this fact a dynamic system whose state ρ closely approximates the sensitivity vector is used instead. This approximation to the sensitivity function has been used by Kreindler [6] and has been shown to give good performance for deterministic systems.

We consider the problem of designing feedback control laws for a class of multi-input-multi-output discrete time stochastic systems with unknown parameters. The performance index to be minimized is quadratic in the state and the control. The optimization is to be performed so that a measure of the energy the control spends in estimation does not exceed the budget available for estimation. We model cost of estimation as a quadratic function of the state sensitivity functions. We seek a feedback solution for the control that explicitly takes

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into account the accuracy of the estimation and that distributes the estimation effort in an optimal fashion.

2. Problem Formulation

Consider the discrete linear system

$$x_{k+1} = Ax_k + Bu_k + w_k \qquad (1)$$
$$y_k = x_k$$

where $x_k \in R^n$, $u_k \in R^m$, $y_k \in R^n$, and w_k is normal with zero mean and covariance V_w. A and B are unknown random matrices of appropriate dimension and might be time varying. The state and the disturbance at the same instant are statistically independent. The entries of A and B are considered independent. The performance index to be minimized is:

$$J_1 = E\{\sum_{k=N_o}^{N_o+\nu-1} x_k'Qx_k + u_k'Ru_k\} \qquad (2)$$

subject to

$$J_2 = E\{\sum_{k=N_o}^{N_o+\nu-1} \rho'(k)W^{N_o}\rho(k)\} \le r \qquad (3)$$

where

$$\rho(k) = (\rho_1'(k),\ldots,\rho_j'(k),\ldots,\rho_d'(k))',$$

and ρ_j which is designed to closely approximate the state sensitivity, satisfies the following equation:

$$\rho_j(k+1) = (A_{\theta_j} - B_{\theta_j}K_1(k))x_k + (\hat{A}-\hat{B}K_1(k))\rho_j(k) - \sum_{i=1}^{d} B_{\theta_j}K_{2i}(k)\rho_i(k) \qquad (4)$$

where A_{θ_j} and B_{θ_j} are the derivatives of A and B with respect to θ_j, $K_1(k)$ and $K_2(k)$ are matrices to be found in the algorithm and $\hat{A},\hat{B}$ are the latest estimates of A and B respectively. θ_j is a component of the vector θ which is formed by the unknown entries of the matrices A and B taken by rows. W^{N_o} is an unknown diagonal matrix to be chosen by the designer at time N_o and satisfies $W_i^{N_o} \ge \epsilon_i \ge 0$ for $i = 1,\ldots,nd$ and $\mathrm{tr}\, W^{N_o} = 1$. The nonnegative numbers ϵ_i are the specified minimum relative weights where $\sum_{i=1}^{nd}\epsilon_i \le 1$. Q is an $n\times n$ positive semidefinite matrix and R is an $m\times m$ positive definite matrix. Both Q and R may vary with k.

The choice of the matrix W^{N_o} allows us to distribute our estimation effort. According to the Cramer-Rao inequality for better estimation a *maximization of the sensitivity with respect to* the unknown parameters to be estimated is required. Then we choose the matrix W^{N_o} such that the large state sensitivities are kept large in order to estimate those parameters more accurately. The design of W^{N_o} will be accomplished as follows: *The sensitivity constraint* (3) is appended through a Lagrange multiplier to the cost functional J_1 and the cost function is minimized with respect to the control law u_k and the weighting matrix W^{N_o}. Through the bound r and the design of the weighting matrix W^{N_o} as indicated, the following goals can be achieved:

i) The larger sensitivity terms in (3) will receive less weight so that they will be less affected by the control than the smaller sensitivity terms which will receive more weight. Thus the control will increase the accuracy to which the parameters that affect the state more at the expense of a decrease in the accuracy of the parameters that affect the system state less.

ii) The estimation effort of the control will be rationally distributed to estimate better the parameters that have greater influence on the state.

We restate our original problem as follows: consider the augmented system

$$\xi_{k+1} = \tilde{A}_k\xi_k + \tilde{B}_k u_k + \Gamma w_k \qquad (5)$$

with perfect state information. We want to *minimize with respect to* u_k *and* W^{N_o} *the performance index*:

$$J = E\{\sum_{k=N_o}^{N_o+\nu-1} \xi_k'\tilde{Q}_k\xi_k + u_k'Ru_k\} \qquad (6)$$

where

$$\tilde{A}_k = \begin{bmatrix} A & 0 & 0 & \cdots & 0 \\ [A_{\theta_1} - B_{\theta_1}K_1(k)] & [\hat{A}-\hat{B}K_1(k) - B_{\theta_1}K_{21}(k)] & -B_{\theta_1}K_{22}(k) & \cdots & -B_{\theta_1}K_{2d}(k) \\ \vdots & & & & \vdots \\ [A_{\theta_d} - B_{\theta_d}K_1(k)] & -B_{\theta_d}K_{21}(k) & -B_{\theta_d}K_{22}(k) & \cdots & [\hat{A}-\hat{B}K_1(k) - B_{\theta_d}K_{2d}(k)] \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} B \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} I_{n\times n} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \tilde{Q}_k = \begin{bmatrix} Q & 0 \\ 0 & \lambda W^{N_o} \end{bmatrix}$$

$\tilde{A}$ and $\tilde{Q}$ are $[(d+1)n]^2$ matrices, $\tilde{B}$ is a $(d+1)n\times m$ matrix, Γ is a $(d+1)n\times n$ matrix and w_k is of dimension n.

The problem formulation in (5) and (6) was obtained by appending J_2 through a Lagrange multiplier λ to the cost function J_1. The Lagrange multiplier λ is chosen so that (3) is satisfied.

3. Solution

The optimal solution of problem (6) subject to (5) is extremely involved and complicated; to seek a closed loop control law would be impractical. A suboptimal solution which is of the feedback form, is sought. First we will find a control law for a fixed W^{N_o}. Next we will optimize with respect to W^{N_o}.

3.1. Computation of u_{N_o}

To find the control u_{N_o} given the information up to N_o and W^{N_o} we proceed according to the following steps:

a) Compute the estimates $\hat{A}_{N_o}$ and $\hat{B}_{N_o}$ based on the information received up to N_o.

b) Apply the certainty equivalence principle to the augmented system. The plant matrices are assumed to be $\tilde{A}_{N_o}, \tilde{B}_{N_o}$. Although this will lead to a certainty equivalent controller in the augmented state space, the error in the estimates is incorporated through the sensitivity constraint and the controller will be cautious. The application of the certainty equivalence principle to the augmented system will result in a better control performance than the certainty equivalent controller for the original problem because the uncertainty of the parameters can be influenced through the sensitivity constraint. The solution will differ from the original certainty equivalent controller because the system matrix $\tilde{A}^{N_o}$ contains unknown feedback gains.

The suboptimal feedback solution outlined in the above steps is given in Theorem 1.

Theorem 1: For a fixed W^{N_o} and given $\hat{A}_{N_o}$ and $\hat{B}_{N_o}$, if equation (9) admits a solution, the control u_{N_o} that minimizes (6) is:

$$u_{N_o} = -(R+\tilde{B}'^{N_o}_{N_o} P^{N_o}_{N_o+1} \tilde{B}^{N_o}_{N_o})^{-1} (\tilde{A}'^{N_o}_{N_o} P^{N_o}_{N_o+1} \tilde{B}^{N_o}_{N_o})' \xi_{N_o} \tag{7}$$

where $P_i^{N_o}$ is the solution to

$$P_i^{N_o} = \tilde{Q}_i + \tilde{A}'^{N_o}_i (K_i) P^{N_o}_{i+1} \tilde{A}^{N_o}_i (K_i) - (\tilde{A}'^{N_o}_i (K_i) P^{N_o}_{i+1} \tilde{B}^{N_o}_i)(R + \tilde{B}'^{N_o}_i P^{N_o}_{i+1} \tilde{B}^{N_o}_i)^{-1} (\tilde{A}'^{N_o}_i (K_i) P^{N_o}_{i+1} \tilde{B}^{N_o}_i)' \tag{8}$$

$$P^{N_o}_{N_o+\nu-1} = \tilde{Q}_{N_o+\nu-1} \qquad i = N_o, \ldots, N_o+\nu-2$$

and

$$K_i = (R+\tilde{B}'^{N_o}_i P^{N_o}_{i+1} \tilde{B}^{N_o}_i)^{-1} (\tilde{A}'^{N_o}_i (K_i) P^{N_o}_{i+1} \tilde{B}^{N_o}_i)' \tag{9}$$

$$K_{N_o+\nu-1} = 0 \qquad i = N_o, \ldots, N_o+\nu-2$$

where $K_i = [K_1(i) \vdots K_{21}(i) \vdots \cdots \vdots K_{2d}(i)]$,

$$\tilde{A}_k^{N_o} = \begin{bmatrix} \hat{A}_{N_o} & 0 & 0 & 0 \\ [A_{\theta_1} - B_{\theta_1} K_1(k)] & [\hat{A}_{N_o} - \hat{B}_{N_o} K_1(k) - B_{\theta_1} K_{21}(k)] & -B_{\theta_1} K_{22}(k) \ldots & -B_{\theta_1} K_{2d}(k) \\ \vdots & & & \\ [A_{\theta_d} - B_{\theta_d} K_1(k)] & -B_{\theta_d} K_{21}(k) & \ldots & [\hat{A}_{N_o} - B_{N_o} K_1 - B_{\theta_d} K_{2d}(k)] \end{bmatrix}$$

$$\tilde{B}_k^{N_o} = [\hat{B}'^{N_o}_k \vdots \underline{0} \ldots \underline{0}]'.$$

$\hat{A}_{N_o}$ and $\hat{B}_{N_o}$ are the estimates of A and B given the information up to N_o.

Proof: To find u_{N_o}, K_{1N_o} and K_{2N_o} we apply the dynamic programming algorithm and find the control law that minimizes the cost to go at every k. For details of the proof refer to reference [9].

In Theorem 1 the certainty equivalent controller for the enlarged system was found. Note that the matrix $\tilde{A}_k$ in equation (5) depends on the matrices $K_1(k)$ and $K_2(k)$ which are found through the design of the control u_k.

The existence of the solution of (7) will depend on whether (9) which is linear in K_i admits a solution. This question is addressed in Theorem 2.

Theorem 2: The rank condition:

$$\text{rank}\{I_{[n(d+1)]^2} \otimes (I - F_i M) + \sum_{j=1}^{d} T_j' \otimes (F_i S_j \hat{B}_{N_o})\} =$$

$$\text{rank}\{I_{[n(d+1)]^2} \otimes (I - F_i M) + \sum_{j=1}^{d} T_j' \otimes (F_i S_j \hat{B}_{N_o})), \; f(F_i \tilde{\tilde{A}}_i^{N_o})\},$$

where $f(F_i \tilde{\tilde{A}}_i^{N_o})$ are the columns of the matrix $F_i \tilde{\tilde{A}}_i^{N_o}$ *written as a long vector, is necessary and* sufficient for the existence of a solution to equation (9), where

$$F_i = (R+\tilde{B}'^{N_o}_i P^{N_o}_{i+1} \tilde{B}^{N_o}_i)^{-1} \tilde{B}'^{N_o}_i P^{N_o}_{i+1}, \tag{10}$$

where

$$S_j = \begin{bmatrix} 0 \\ \vdots \\ I_{n\times n} \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \\ \\ \leftarrow j+1 \\ \\ n(d+1)\times n \end{array}, \quad T_j = \begin{bmatrix} -S_j' \\ \\ 0 \; \dots \; 0 \\ \\ 0 \; \dots \; 0 \end{bmatrix}, \; n(d+1)\times n(d+1)$$

$$M = \begin{bmatrix} 0 \\ -B_{\theta_1} \\ \vdots \\ -B_{\theta_d} \end{bmatrix} n(d+1)\times m, \quad \tilde{A}_k^{N_o} = \begin{bmatrix} \hat{A}_{N_o} & 0 & & 0 \\ A_{\theta_1} & \hat{A}_{N_o} & & \\ & & \ddots & \\ A_{\theta_j} & & \hat{A}_{N_o} & 0 \\ & & & \ddots \\ A_{\theta_d} & & & \hat{A}_{N_o} \end{bmatrix}.$$

Proof:

i) Rewrite equation (9) as the equivalent equation:

$$(I-F_iM)K_i - F_i \sum_{j=1}^{d} S_j \hat{B}_{N_o} K_i T_j = F_i \tilde{A}_i^{N_o} \qquad (11)$$

Equation (11) is obtained after manipulating equation (9) and identifying terms.

ii) Write equation (11) as a vector equation in the columns of K_i. This results in the vector equation,

$$(I \otimes (I-F_iM) + \sum_{i=1}^{d} T_j' \otimes (F_i S_j \hat{B}_{N_o}))k_i = f(F_i \tilde{A}_i^{N_o}) \qquad (12)$$

where k_i are the columns of K_i written as a long column vector.

From equation (12) the rank condition given in Theorem 2 follows.

3.2. Computation of W^{N_o}

In the subsection above we found the feedback solution u_{N_o} for a given matrix W^{N_o}, for a given control sequence u_i, $i = N_o, \dots, N_o + \nu - 1$.

Given the information up to N_o, compute the estimate $\hat{A}_{N_o}$ and $\hat{B}_{N_o}$. We choose a fixed sequence u_i, $i = N_o, \dots, N_o + \nu - 1$ by the method of subsection 3.1,

$$u_i = -(R + \tilde{B}_i'^{N_o} P_{i+1}^{N_o} \tilde{B}_i^{N_o})^{-1} (\tilde{A}_i'^{N_o} P_{i+1}^{N_o} \tilde{B}_i^{N_o})' \xi_i \qquad (13)$$

where $P_i^{N_o}$ is given by equations (8) and (9). For a given control sequence, J in (6) is linear in W^{N_o}. We minimize this linear function subject to the linear constraints

$$\begin{aligned} &W_i^{N_o} \geq \epsilon_i, \; i = 1, \dots, nd \\ &\sum_{i=1}^{nd} W_i^{N_o} = 1 \end{aligned} \qquad (14)$$

where $\epsilon_i \geq 0$ *and these numbers are given.* This simple linear programming problem is *easily solved by examining J at the* nd *vertices* of the feasible space

$$\begin{aligned} &W_i^{N_o} = \epsilon_i, \; i = 1, \dots nd, \; i \neq j \\ &W_j^{N_o} = 1 - \sum_{i \neq j} \epsilon_i, \; j = 1, \dots, nd \end{aligned} \qquad (15)$$

and we choose a vertex at which J is a minimum. If this vertex is the same as that assumed to obtain the control sequence u_i, we have the optimum W^{N_o}. If not, we try another vertex, choose another control sequence, and repeat the cycle. If a solution exists, we obtain it in at most nd steps.

After substituting the value of u_i from (13) into (6), the value of J can be written as

$$J_{N_o} = \xi_{N_o}' P_{N_o}^{N_o} \xi_{N_o} + \sum_{J=N_o+1}^{N_o+\nu-1} tr\,(\tilde{Q}_j P_{\xi_j}^{N_o}) \qquad (16)$$

where $P_{\xi_j}^{N_o}$ is the covariance of ξ_j given the information up to time N_o. It satisfies the propagation equation

$$\begin{aligned} &P_{\xi_{j+1}}^{N_o} = \tilde{A}_{fj}^{N_o} P_{\xi_j}^{N_o} \tilde{A}_{fj}'^{N_o} + \Gamma V_w \Gamma', \; j = N_o, \dots, N_o + \nu - 1. \\ &P_{\xi_{N_o}}^{N_o} = 0 \end{aligned} \qquad (17)$$

The matrix $\tilde{A}_{fj}^{N_o}$ is the same as $\tilde{A}_j^{N_o}$ except that instead of the first row

$[A \quad 0 \quad \dots \quad 0]$

we have

$[A-BK_1 \quad -BK_{21} \quad -BK_{22} \quad \dots \quad -BK_{2d}]$

and A and B are evaluated as $\hat{A}_{N_o}$ and $\hat{B}_{N_o}$. For details see [9]. For a fixed control sequence, $P_{\xi}^{N_o}$ is fixed and in minimizing J_N in (16) with respect to W^{N_o} only $\tilde{Q}_{j_N}$ and $P_N^{N_o}$ vary with W^{N_o}, and they are linear in W^{N_o}. The dependence of $P_i^{N_o}$ on W^{N_o} in i is only through $\tilde{Q}_i$. That is, $\tilde{A}$ and $\tilde{B}$ are fixed when (8) is used to generate $P_{N_o}^{N_o}$.

The solution obtained above is open loop optimal for the augmented system for a given estimate $\hat{A}_{N_o}$ and $\hat{B}_{N_o}$. We use the feedback gains K_{1N_o} and K_{2N_o} for implementation in feedback form at time N_o. As new information is received one time unit later, the value of N_o is increased by one and the process is repeated to obtain the next feedback gain values. This is essentially an open loop optimal feedback solution.

The algorithm presented above will automatically assign more weight to the small state sensitivity functions and less weight to the large state sensitivity functions; this is so because the estimation budget is fixed and the optimality condition for W has to be satisfied. Since W is the weighting matrix of ρ which is designed to closely approximate the state sensitivity, and W^{N_o} is found to minimize the performance index J in equation (6), then the components of W^{N_o} will be found such that less weight will be assigned to large state sensitivity functions and more weight will be assigned to small state sensitivity functions.

3.3. Sensitivity Approximations

In this section we will analyze the sensitivity approximation used in the development of the previous section. Consider the approximate feedback sensitivity [6] given by (4) and reproduced here:

$$\rho_j(k+1) = (A_{\theta_j} - B_{\theta_j} K_1(k))x_k + (\hat{A}-\hat{B}K_1(k))\rho_j(k) - \sum_{i=1}^{d} B_{\theta_j} K_{2i}(k)\rho_i(k) \tag{18}$$

$i,j = 1,\ldots,d$, where ρ_j depends on K_1 and K_{2i}, which are the feedback gains that define the feedback control law obtained from the algorithm.

The exact sensitivity functions for the feedback system are given by

$$\sigma_j(k+1) = (\hat{A}-\hat{B}K_1(k))\sigma_j(k) + (A_{\theta_j} - B_{\theta_j}K_1(k))x_k - \sum_{i=1}^{d}(B_{\theta_j}K_{2i}(k)\rho_i(k) + \hat{B}K_{2i}\frac{\partial\rho_i(k)}{\partial\theta_j}) \tag{19}$$

and

$$\frac{\partial\rho_i(k+1)}{\partial\theta_j} = (\hat{A}-\hat{B}K_1(k))\frac{\partial\rho_i(k)}{\partial\theta_j} + (A_{\theta_j} - B_{\theta_j}K_1(k))\sigma_k - \sum_{i=1}^{d} B_{\theta_j}K_{2i}(k)\frac{\partial\rho_i(k)}{\partial\theta_j}. \tag{20}$$

The true feedback sensitivity σ will be the solution of four sets of simultaneous equations, (1), (18), (19), and (20) using the mean values of A and B in equation (1).

If we compare (18) with (19) by writing an error equation, we see that ρ will approximate σ closely if the first derivatives of ρ with respect to θ_j are very small and if $\hat{A}-\hat{B}K$ is stable. This can be seen through the following error equation:

$$e_j(k+1) = (\hat{A}-\hat{B}K_1(k))e_j(k) - \sum_{i=1}^{d}\hat{B}K_{2i}\frac{\partial\rho_i(k)}{\partial\theta_j}. \tag{21}$$

4. Example

We consider the system

$$x_{k+1} = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_k \tag{22}$$

$$y_k = x_k$$

where $a = (a_1 \;\; a_2)$ are random variables with prior statistics, $E\left\{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right\} = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$, $V_a = \mathrm{Diag}(0.0001, 0.0001)$. The noise w_k is a random sequence with zero mean and variance 1.0. The initial state x_{N_o} is normal with mean $\bar{x}_{N_o} = (5 \;\; 5)'$ and variance $V_{x_{N_o}} = \mathrm{Diag}(0.0001, 0.0001)$.

We seek a control law u_k, and matrix W^{N_o} such that the following performance index is minimized,

$$J_{N_o} = E\{\sum_{k=N_o}^{N_o+\nu-1} x_k'Qx_k + u_k'Ru_k\} \tag{23}$$

subject to

$$E\{\sum_{k=N_o}^{N_o+\nu-1} \rho_k' W^{N_o}\rho_k\} \le r \tag{24}$$

$$\sum_{i=1}^{4} W_i^{N_o} = 1.0, \quad \forall k$$

where $W_i^{N_o}$ are the components of the diagonal W^{N_o}. In this example we take $\nu = 4$, $Q = \mathrm{Diag}(1.0, 1.0)$, $R = 1.0$. The enlarged system $\begin{bmatrix} x_k \\ \rho_k \end{bmatrix}$ is given by

$$\begin{bmatrix} x_{k+k} \\ \rho_{1k+1} \\ \rho_{2k+1} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ A_{\theta_1} & A_{N_o} - BK_1 & 0 \\ A_{\theta_2} & 0 & A_{N_o} - BK_1 \end{bmatrix} \begin{bmatrix} x_k \\ \rho_{1k} \\ \rho_{2k} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u_k + \begin{bmatrix} \Gamma_w \\ 0 \\ 0 \end{bmatrix} w_k \tag{25}$$

where $\theta = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, $\Gamma_w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, A_{N_o} is the matrix A with a_1 and a_2 replaced by their means at N_o, and K is the feedback control gain which is designed to achieve a close approximation of the state sensitivity by ρ_j.

Applying the certainty equivalent principle to system (22) we obtain $J_{CE} = 80.9$. Using the sensitivity approach, $\bar{J} = 52.71$, which is a good improvement in the cost. From Table 1 we see that the assignment of the weighting matrix W^{N_o} is such that the smallest sensitivity functions receive weight 1.0 and the others weight zero. Thus the theoretical goal is achieved. This is so because we wanted to keep the large sensitivity functions as large as possible so that the parameters whose

effect on the state of the system is large are estimated as accurately as possible. Also the control u is affected by the choice of W^{N_o} and by the feedback of the sensitivity functions in such a way that the control performance is improved.

6. Conclusion

In this paper we described a feedback control law that explicitly takes into account the accuracy of estimation and which distributes the estimation effort in an optimal fashion through the design of the weighting matrix W. This is achieved by incorporating the estimation cost J_2 and designing the weighting matrix W for the sensitivity terms in J_2 in such a way that all the energy available for estimation is used. Moreover through the constrained minimization with respect to W the available estimation energy is distributed so that the larger sensitivity terms receive less weight and the smaller sensitivity terms more weight, thereby forcing the control to affect the accuracy of estimation in such a way that the more crucial parameters are estimated more accurately.

7. References

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Table 1. Average Trajectory of the feedback gains K, control law, state, approximate sensitivity and weighting matrix W.

	$N_o=1$	$N_o=2$	$N_o=3$	$N_o=4$
K_{11}	0.38170	0.34616	0.38567	-0.255232
K_{12}	-0.22056	-0.26387	-0.26391	-0.25532
$K_{21,1}$	0.0	0.0	0.0	0.01703
$K_{21,2}$	0.0	0.0	0.0	-0.00904
$K_{22,1}$	-0.00686	-0.01534	-0.01540	0.0284
$K_{22,2}$	0.04218	0.03760	0.03767	-0.01076
u	-0.80410	-1.50834	-0.03591	-0.35218
W_{N_o}	Diag(0.0,0.0,1.0, 0.0)	Diag(0.0,0.0,0.0, 1.0)	Diag(0.0,0.0,0.0, 1.0)	Diag(0.0,0.0,0.0, 1.0)
X_1	5.0016445	0.60718	0.6284	0.02516
X_2	0.60718	0.6284	-0.06291	-0.26226
ρ_{11}	0.0	4.99672	5.32945	1.53338
ρ_{12}	4.99673	5.29231	1.53385	1.25102
ρ_{21}	0.0	5.00164	1.47897	1.27078
ρ_{22}	5.0016445	0.56938	1.08969	0.29428

Average cost $\bar{J}$ = 52.7053

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A stochastic adaptive control problem that incorporates estimation cost considerations is formulated. A sensitivity index is introduced to represent the estimation cost. The control effort on estimation is distributed according to the accuracy required to achieve a given control objective when there is a bound on the estimation cost. The effect of the control on the sensitivity functions is used to influence the future uncertainties of the system. A dynamic feedback control algorithm is proposed that explicitly takes into account the accuracy of estimation and distributes the estimation effort in an optimal fashion.

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